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Efficiency of Differential Transform Method to Solve Systems of Ordinary Differential Equations

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Abstract: **In this paper, the differential transform method is employed to construct the exact solution for systems of ordinary differential equations. It has been observed that the proposed method is very efficient and reliable for the solution of systems of ordinary differential equations. Numerical results represent the effectiveness and efficiency of the proposed method.The differential transformation method (DTM) is used to compute an approximation to the solution of the systems of ordinary differential equations. The results are compared with the results obtained by different numerical methods. Some plots are presented to show the reliability and simplicity of the method.**

Keywords: **Differential Transform, Series solution, systems of ordinary differential equations.**

I. INTRODUCTION

In this paper we apply DTM (one dimensional) on linear differential equation on some example and the result obtained by it are compared with the result obtain by Laplace transform method which are exact solutions. In recent years, Abdel-Halim Hassan [3] used differential transform method to solve this type of equations. Arikoglu A applied DTM to obtain numerical solution of differential equations[4]. Ayaz F has used DTM to find the series solution of system of differential equations[5]. Bert.W has applied DTM on system of linear equation and analysis its solutions[6]. Chen used DTM to obtain the solutions of nonlinear system of differential equations[7]. Chen C.L. has applied DTM technique for steady nonlinear heat conduction problems[8]. Duan Y used DTM for Burger's equation to obtain the series solution[9]. The DTM have find out series solution and that solution compared with decomposition method for linear and nonlinear initial value problems and prove that DTM is reliable tool to find the numerical solutions [2,10]. Kuo B has been used to find the numerical solution of the solutions of the free convection Problem[12]. Montri Thongmoon has been used to find the numerical solution of ordinary differential equations[13]. The concept of Differential Transform Method (DTM) was first proposed by Zhou and proves that DTM is an iterative procedure for obtaining analytic Taylor's series solution of differential equations. DTM is very useful to solve equation in ordinary differential equation. It is also applied to solve boundary value problems[14]. The method also used to solve integral equations [1]. The basic idea of the DTM was introduced by Zhou [3].In what following we introduce a few notations for the DTM. Suppose that the solution u(x, t) is analytic at U(X,T), then the solution u(x, t) can be represented by the Taylor series[1].

$$u(x,t)=\sum_{k_1=0}^{\infty}\dots\sum_{k_n=0}^{\infty}\sum_{h=0}^{\infty}\frac{1}{k_1!\dots k_n!\,h!}\left[\frac{\partial^{k_1+\dots k_n+h}u(x^{\sim},t^{\sim})}{\partial x_1^{k_1}\dots\partial x_n^{k_n}\,\partial t^h}\right]\left(\prod_{i=1}^{n}(x_i-x_i^{\sim})^{k_i}\right)(t-t^{\sim})^h \qquad (1)$$

Definition 1.1

Let us define the (n + 1) dimensional differential transform U(k, h) of u(x,t) at $(x^{\sim},t^{\sim})$ by

$$U(k,h)=\frac{1}{k_1!\dots k_n!h!}\left[\frac{\partial^{k_1+\dots k_n+h}u(x^{\sim},t^{\sim})}{\partial x_1^{k_1}\dots\partial x_n^{k_n}\,\partial t^h}\right] \qquad (2)$$

Definition 1.2

The differential inverse transform of U (k, h) is defined by u(x, t) of the form in (1). Thus, u(x, t) can be written by

$$u(x,t) = \sum_{k_1=0}^{\infty} \dots \sum_{k_n=0}^{\infty} \sum_{h=0}^{\infty} \mathrm{U(k,h)} \ \left(\prod_{i=1}^{n} (x_i - x_i^{\sim})^{k_i}\right)(t - t^{\sim})^{h} \tag{3}$$

An arbitrary function $f(x)$ can be expanded in Taylor series about appoint $x = 0$ as:

$$f(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!} \left\{\frac{d^k f}{dx^k}\right\}_{x=0} \tag{4}$$

The differential transformation of $f(x)$ is defined as:

$$F(k) = \frac{1}{k!} \left\{\frac{d^k f(x)}{dx^k}\right\}_{x=0} \tag{5}$$

Then the inverse differential transform is:

$$f(x) = \sum_{k=0}^{\infty} x^k F(k) \tag{6}$$

II. THE FUNDAMENTAL OPERATION OF DIFFERENTIAL TRANSFORMATION METHOD[1,4]

(2.1) If $y(x) = g(x) \pm h(x) then Y(k) = G(k) \pm H(k)$

$$F(k) = \frac{1}{k!} \left[\frac{d^k f}{dx^k}\right]_{x=0} = \frac{1}{k!} \left\{\frac{d^k y(x)}{dx^k}\right\}_{x=0}$$

$$= \frac{1}{k!} \left\{\frac{d^k (g(x) + \mathrm{h(x)})}{dx^k}\right\}_{x=0}$$

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$$= \frac{1}{k!} \left\{\frac{d^k g(x)}{dx^k} \pm \frac{d^k h(x)}{dx^k}\right\}_{x=0} = \frac{1}{k!} \left\{\frac{d^k g(x)}{dx^k}\right\}_{x=0} \pm \frac{1}{k!} \left\{\frac{d^k h(x)}{dx^k}\right\}_{x=0}$$

$$= G(k) \pm H(k)$$

(2.2) $If y(x) = \propto g(x) then Y(k) = \propto G(k)$

Proved from the definition

(2.3) $y(x) = \frac{dg(x)}{dx} then Y(k) = (k + .1)G(k + 1)$

$$Y(k) = \frac{1}{k!} \left\{\frac{d^k y(x)}{dx^k}\right\}_{x=0} = \frac{1}{k!} \left\{\frac{d^k}{dx^k} \left(\frac{dg(x)}{dx}\right)\right\}_{x=0} = \frac{1}{k!} \left\{\frac{d^{k+1} \mathrm{g(x)}}{dx^{k+1}}\right\}_{x=0}$$

from the definition we have.

$$= \frac{1}{k!} \{(k + 1)!\, G(k + 1)\} = (k + 1)G(k + 1)$$

(2.4) $y(x) = \frac{d^2 g(x)}{dx^2}$ then $Y(k) = (k + 1)(k + 2)G(k + 2)$ Proof like (3)

(2.5) $y(x) = \frac{d^m g(x)}{dx^m}$ then $Y(k) = (k + 1)(k + 2) \dots (k + m)G(k + m)$

(2.6) $y(x) = 1 \implies Y(k) = \delta(k)$

(2.7) $y(x) = x \implies Y(k) = \delta(k - 1)$

$$Y(k) = \frac{1}{k!} \left\{\frac{d^k x}{dx^k}\right\} = \delta(k)$$

(2.8) $y(x) = x^m \implies Y(k) = \delta(k - m) = \begin{Bmatrix} 1 & \mathrm{k = m} \\ 0 & \mathrm{k \neq m} \end{Bmatrix}$

(2.9) $y(x) = g(x)h(x) then Y(k) = \sum_{m=0}^{k} H(m)G(k - m)$

$$Y(k) = \frac{1}{k!}\left\{\frac{d^k g(x)h(x)}{dx_k}\right\}_{x=0} \quad ,Let \ \ k = 1$$

$$Y(k) = \left\{\frac{dgh}{dx}\right\}_{x=0} = \left\{g(x)\frac{dh}{dx} + h(x)\frac{dg}{dx}\right\}_{x=0}$$

$$= g(0)\left\{\frac{dh}{dx}\right\}_{x=0} + h(0)\left\{\frac{dg}{dx}\right\}_{x=0}$$

$$G(0)H(1-0) + h(0)G(1-0)$$

$$letk = 2$$

$$Y(k) = \frac{1}{2!}\left\{\frac{d^2 gh}{dx^2}\right\}_{x=0} = \frac{1}{2!}\left\{\frac{d}{dx}\left(\frac{d}{dx}gh\right)\right\}_{x=0}$$

$$= \frac{1}{2!}\left\{\frac{d}{dx}\left(g\frac{dh}{dx} + h\frac{dg}{dx}\right)\right\}_{x=0}$$

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$$= \frac{1}{2!}\left\{\left(g\frac{d^2h}{dx^2} + \frac{dh}{dx}\frac{dg}{dx} + h\frac{d^2g}{dx^2} + \frac{dg}{dx}\frac{dh}{dx}\right)\right\}_{x=0}$$

$$= G(0)H(2-0) + H(1-0)G(1-0) + H(0)G(2-0)$$

$$= G(0)H(2) + H(1)G(1) + H(0)G(2)$$

in general we have

$$Y(k) = \sum_{m=0}^{k} H(m)G(k-m)$$

(2.10) $y(x) = e^{\lambda x} thenY(k) = \frac{\lambda^k}{k!}$

$$Y(k) = \frac{1}{K!}\left\{\frac{d^k e^{\lambda x}}{dx^k}\right\} = \frac{1}{K!}\left\{\lambda^k e^{\lambda x}\right\}_{x=0} = \frac{\lambda^k}{k!}$$

(2.11) $y(x) = (1+x)^m \quad ,Y(k) = \frac{m(m-1)\ldots(m-k+1)}{k!}$

$$Y(k) = \frac{1}{K!}\left\{\frac{d^k(1+x)^m}{dx^k}\right\} = \frac{m(m-1)(m-2)\ldots(m-k+1)}{k!}$$

(2.12) $y(X) = \sin(wx + \alpha)$, then

$$Y(k) = \frac{w^k}{k!}\sin(\frac{k\pi}{2}+\alpha)$$

where w and αconstants.

(2.13) $y(x) = \cos(wx + \alpha)$, then

$$Y(k) = \frac{w^k}{k!}\cos(\frac{k\pi}{2}+\alpha),$$

where w and α constants.

III. SYSTEMS OF ORDINARYDIFFERENTIAL EQUATIONS

Discussed the system of differential equation can arise from a population problem in which we keep track of the population of both the prey and the predator .It makes sense that the number of prey present will affect the number of the predator present. likewise, the number of predator present will affect the number of prey present. therefor the differential equation that governs the population of either the prey or the predator should in some way depend on the population of other. This will lead to tow differential equations that must be solved simultaneously in order to determine the population of the prey and the predator. The whole point of this is to notice that systems of differential equations can arise quite easily from naturally occurring situations.

The standard form of system of ordinary differential equations of then order with conditions is considered [6]as

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$$\emptyset_1\big(x, y_1(x), y_1'(x), \dots, y_1^{(n)}(x), y_2(x), y_2'(x), \dots, y_2^{(n)}(x), \dots, y_N(x), y_N'(x), \dots, y_N^{(n)}(x)\big) = 0$$

$$\emptyset_2\left(x, y_1(x), y_1'(x), \dots, y_1^{(n)}(x), y_2(x), y_2'(x), \dots y_2^{(n)}(x), \dots, y_N(x), y_N'(x), \dots, y_N^{(n)}(x)\right) = 0$$

$\vdots$

$$\emptyset_N\big(x, y_1(x), y_1'(x), \dots, y_1^{(n)}(x), y_2(x), y_2'(x), \dots, y_2^{(n)}(x), y_N(x), y_N'(x), \dots, y_N^{(n)}(x)\big) = 0 \qquad (7)$$

With initial values given for.

$$y_1(x_0) = y_{10}, y_1'(x_0) = y'_{10}, \dots, y_1^{(n-1)}(x_0) = y_{10}^{(n-1)}$$

$$y_2(x_0) = y_{20}, y_2'(x_0) = y_{20}', \dots, y_2^{(n-1)}(x_0) = y_{20}^{(n-1)} \qquad (8)$$

Where $\emptyset_1, \emptyset_2, \cdots\cdots\cdots \emptyset_N$ are nonlinear continuous function .

In this paper we will derive the (DTM) that we have developed for the approximate solution for y(x)= $(y_1(x), y_2(x), \dots\dots, y_N(x))$.

IV. APPLICATIONS AND NUMERICAL RESULTS

In order to illustrate the advantages and the accuracy of the (DTM) for solving systems of ordinary differential equations

4.1 Example 1

Assume the following system of ordinary differential equations :

$$\left.\begin{aligned} y_1'(x) &= 3y_1(x) - y_2(x) + y_3(x) \\ y_2'(x) &= 2y_1(x) + y_3(x) \\ y_3'(x) &= y_1(x) - y_2(x) + 2y_3(x) \end{aligned}\right\} \qquad (9)$$

with the conditions:

$$y_1(0) = 0, y_2(0) = 0 \;\;,\;\; y_3(0) = 1\,\} \qquad (10)$$

then applying DTM: equation (9) of (10) are transformed as follows:

$$\left.\begin{aligned} Y_1(k+1) &= \tfrac{1}{k+1}[3Y_1(k) - Y_2(k) + Y_3(k)] \\ Y_2(k+1) &= \tfrac{1}{k+1}[2Y_1(k) + Y_3(k)] \\ Y_3(k+1) &= \tfrac{1}{k+1}[Y_1(k) - Y_2(k) + 2Y_3(k)] \end{aligned}\right\} \qquad (11)$$

$$Y_1(0) = 0\,, Y_2(0) = 0\,, Y_3(0) = 1\,\} \qquad (12)$$

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thus:

If k =0 then $Y_1(1) = 1$, $Y_2(1) = 1$, $Y_3(1) = 2$

If k = 1 then $Y_1(2) = 2$, $Y_2(2) = 2$,$Y_3(2) = 2$

If k = 2 then $Y_1(3) = 2$,$Y_2(3) = 2$, $Y_3(3) = \frac{4}{3}$

If k = 3 then $Y_1(4) = \frac{4}{3}$, $Y_2(4) = \frac{4}{3}$, $Y_3(4) = \frac{2}{3}$

If k = 4 then $Y_1(5) = \frac{2}{3}$, $Y_2(5) = \frac{2}{3}$, $Y_3(5) = \frac{4}{15}$

⋮

therefore the solution equation is given:

$y_i(x) = \sum_{k=0}^{\infty} x^k Y(k)$ for I = 1 ,2 , 3

$$y_1(x) = x + 2x^2 + 2x^3 + \frac{4}{3}x^4 + \frac{2}{3}x^5 \ldots = \mathrm{x}(1 + 2x \quad + \frac{4}{2!}x^2 + \frac{8}{3!}x^3 + \frac{16}{4!}x^4 + \cdots$$

$$y_1(x) = xe^{2x} \qquad (13)$$

$$y_2(x) = x + 2x^2 + 2x^3 + \frac{4}{3}x^4 + \frac{2}{3}x^5 \ldots = xe^{2x} \qquad (14)$$

$$y_3(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5 \cdots = 1 + 2x \quad + \frac{4}{2!}x^2 + \frac{8}{3!}x^3 + \frac{16}{4!}x^4 + \frac{32}{5!}x^5 \ldots = e^{2x} \qquad (15)$$

4.2Example 2

Consider the following system of ordinary differential equation :

$$y'1(x) = y1(x) + y2(x) + e^{-x},\ y'2(x) = y2(x)\} \qquad (16)$$

with the conditions

$$y1(0) = -1\ , y2(0) = 1\} \qquad (17)$$

then applying DTM: equation (16) of (17)

are trans formed as follows:

$$(k+1)Y_1(k+1) - Y_1(k) - Y_2(k) + \frac{1}{k!} = 0 \qquad (18)$$

$(k+1)Y_2(k+1) - Y_2(k) = 0$ (19)

$$Y_1(0) = -1 \quad , Y_2(0) = 1 \qquad (20)$$

Consequently :

When k = 0 then $Y_1(1) = -1$, $Y_2(1) = 1$

When k = 1 then $Y_1(2) = \frac{-1}{2!}$,$Y_2(2) = \frac{1}{2!}$

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When k = 2 then $Y_1(3) = \frac{-1}{3!}$, $Y_2(3) = \frac{1}{3!}$

When k = 3 then $Y_1(4) = \frac{-1}{4!}$, $Y_2(4) = \frac{1}{4!}$

When k =4 then $Y_1(5) = \frac{-1}{5!}$,$(5) = \frac{1}{5!}$

⋮

So the solution is

$$y_i(x) = \sum_{k=0}^{\infty} x^k Y_i(k) \ for\ i = 1.2$$

$y_1(x) = -1 - x - \frac{1}{2!}x^2 - \frac{1}{3!}x^3 - \frac{1}{4!}x^4 - \frac{1}{5!}x^5 \cdots$

$$y_1(x) = -(1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 \ldots = -e^x \qquad (21)$$

$$y_2(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 \cdots = e^x \qquad (22)$$

4.3 Example 3

Assume the following system of ordinary differential equations:

$y'_1(\mathrm{x}) = y_2(x)$(23)

$$y'_2(\mathrm{x}) = -y_1(x) \tag{24}$$

With the conditions:

$$y_1(0) = 1 \,, y_2(0) = 0 \;\} \tag{25}$$

Then applying DTM: equation (23),(24) of (25) are transformed as follow:

$$Y_1(k+1) = \frac{1}{k+1}\,[Y_2(k)] \tag{26}$$

$$Y_2(k+1) = \frac{1}{k+1}[-Y_1(k)] \tag{27}$$

$$Y_1(0) = 1 \,, Y_2(0) = 0 \;\} \tag{28}$$

Thus

$$Y_1(1) = 0 \;, \qquad Y_2(1) = -1$$

$$Y_1(2) = \frac{-1}{2!} \;, \qquad Y_2(2) = 0$$

$$Y_1(3) = 0 \,, \qquad Y_2(3) = \frac{1}{3!}$$

$$Y_1(4) = \frac{1}{4!} \,, \qquad Y_2(4) = 0$$

$$Y_1(5) = 0, \qquad Y_2(5) = \frac{-1}{5!}$$

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$$Y_1(6) = \frac{-1}{6!} \,, \qquad Y_2(6) = 0$$

$$\vdots$$

So the solution is

$y_i\,(x) = \sum_{k=0}^{\infty} x^k\, Y_{i(k)}$for I =1,2

$$y_1(x) = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 \cdots = \cos(x), \tag{29}$$

$$y_2(x) = -x + \frac{1}{3!}x^3 - \frac{1}{5!}x^5 + \cdots = -(x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots = -\sin(x). \tag{30}$$

4.4 Example 4

Assume the following system of ordinary differential equations :

$$\left.\begin{aligned} \mathrm{y}_1'(\mathrm{x}) &= 2\mathrm{y}_1(\mathrm{x}) + y_2(\mathrm{x}) + \sin(\mathrm{x}) \\ \mathrm{y}_2'(\mathrm{x}) &= 2y_2(\mathrm{x}) + \cos(x) \end{aligned}\right\} \tag{31}$$

with the conditions:

$$\left.\begin{aligned} \mathrm{y}_1(0) &= 1 \\ \mathrm{y}_2(0) &= -1 \end{aligned}\right\} \tag{32}$$

then applying DTM: equation (31)of (32) are transformed as follow:

$$\left.\begin{aligned} Y_1(k+1) &= \frac{1}{k+1}\left[\, 2Y_1(k) + Y_2(k) + \frac{1}{k!}\sin\left(\frac{k\pi}{2}\right)\right] \\ Y_2(k+1) &= \frac{1}{k+1}\left[2Y_2(k) + \frac{1}{k!}\cos\left(\frac{k\pi}{2}\right)\right] \end{aligned}\right\} \quad (33)$$

$$Y_1(0) = 1\,, Y_2(0) = -1 \;\} \quad (34)$$

Consequently :

If k = 0 then $Y_1(1)$ =1, $Y_2(1) = -1$

If k = 1 then $Y_1(2) = 1$, $Y_2(2) = -1$

If k= 2 then $Y_1(3) = \frac{1}{3}$, $Y_2(3) = \frac{-5}{6}$

If k =3 then $Y_1(4) = \frac{-1}{12}$, $Y_2(4) = \frac{-5}{12}$

If k =4 then $Y_1(5) = \frac{-7}{60}$, $Y_2(5) = \frac{-19}{120}$

⋮

then the solution is given by

$y_i(x) = \sum_{k=0}^{\infty} x^k Y_{i(k)}$ for I =1,2

$y_1(x) = Y_1(0) + Y_1(1)x + Y_1(2)x^2 + Y_1(3)x^3 + Y_1(4)x^4 + Y_1(5)x^5 + \cdots$

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$$y_2(x) = Y_2(0) + Y_2(1)x + Y_2(2)x^2 + Y_2(3)x^3 + Y_2(4)x^4 + Y_2(5)x^5 + \cdots$$

$$y_1(x) = 1 + x + x^2 + \frac{1}{3}x^3 - \frac{1}{12}x^4 - \frac{7}{60}x^5 + \cdots = \quad (35)$$

$$y_2(x) = -1 - x - x^2 - \frac{5}{6}x^3 - \frac{5}{12}x^4 - \frac{9}{120}x^5 \;\ldots = \quad (36)$$

V. CONCLUSION

In this work, we successfully apply the **DTM** to find numerical solutions for linear system of ordinary differential equations. It is observed that **DTM** is an effective and reliable tool for the solution of system of ordinary differential equations. The method gives rapidly converging series solutions. The accuracy of the obtained solution can be improved by taking more terms in the solution. In many cases, the series solutions obtained with **DTM** can be written in exact closed form. The present method reduces the computational difficulties of the other traditional methods and all the calculations can be made simple manipulations. Several examples were tested by applying the **DTM** and the results have shown remarkable performance.

REFERENCES

[1] Abdallah Habila Ali, Shama Alamin Mustafa , Applications Of Differential Transform Method To Integral Equations ,American Journal of Engineering Research (AJER) e-ISSN: 2320-0847 p-ISSN : 2320-0936 Volume-7, Issue-1, pp- 271-276,(2018)

[2] Abdallah Habila Ali, Applications of Differential Transform Method To Initial Value Problems, American Journal of Engineering Research (AJER**)** e-ISSN: 2320-0847 p- ISSN : 2320-0936 Volume-6, Issue-12, pp-365-371

[3] Abdel-Halim Hassan I.H., differential transform technique for solving higher order initial value problems, *Applied Mathematics and Computation*, **154,** 299-311 **(2004)**

[4] Arikoglu A. and Ozkol I., Solution of difference equations by using differential transform method, *pplied Mathematics and Computation*, **174,** 1216-1228 **(2006)**

[5] Ayaz F., Solutions of the system of differential equations by differential transform method, *Applied Mathematics and Computation*, **147,** 547-567 **(2004)**

[6] Bert.W., H. Zeng, Analysis of axial vibration of compound bars by differential transform method, *Journal of Sound and Vibration*, **275,** 641-647 **(2004)**

[7] Chen C.K. and S.S. Chen, Application of the differential transform method to a non-linear conservative system, *Applied Mathematics and Computation*, **154,** 431-441**(2004)**

[8] Chen C.L. and Y.C. Liu, Differential transformation technique for steady nonlinear heat conduction problems, *Appl. Math. Computer*, **95**, 155164 **(1998)**

[9] Duan Y., R. Liu and Y. Jiang, Lattice Boltzmann model for the modified Burger's equation, *Appl. Math. Computer*, **202,** 489-497**(2008)**

[10] HassanI H. Abdel-Halim Comparison differential transformation technique with Adomain decomposition method for linear and nonlinear initial value problems, *Choas Solutions Fractals*, **36(1),**53-65 **(2008)**

[11] Khaled Batiha and Belal Batiha, A New Algorithm for Solving Linear Ordinary Differential Equations, *World Applied Sciences Journal*, **15(12),** 1774-1779**, (2011), ISSN 1818-4952**, IDOSI Publications

[12] Kuo B.L., Applications of the differential transform method to the solutions of the free convection Problem, *Applied Mathematics and Computation*, **165,** 63-79 **(2005)**

[13] Montri Thongmoon, Sasitorn Pusjuso The numerical solutions of differential transform method and the Laplace transforms method for a system of differential equations, *Nonlinear Analysis: HybridSystems*, **4,** 425-431 **(2010)**

[14] Zhou X., Differential Transformation and its Applications for Electrical Circuits, Huazhong University Press, Wuhan,